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Quasi-particle description of strongly interacting matter: Towards a foundation

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Abstract. We confront our quasi-particle model for the equation of state of strongly interacting matter with recent first-principle QCD calculations. In particular, we test its applicability at finite baryon densities by comparing with Taylor expansion coefficients of the pressure for two quark flavours. We outline a chain of approximations starting from the Φ-functional approach to QCD which motivates the quasi-particle picture.

1 Introduction

In the last years, great progress has been made in the numerical evaluation of QCD thermodynamics from first principles (dubbed lattice QCD) even for finite chemical potentials [1–7]. While various perturbative expansions [8–13] fail in describing the thermodynamics of strongly interacting matter in the vicinity of T_c (the (pseudo-) critical temperature of deconfinement and chiral symmetry restauration), different phenomenological approaches exist which aim to reproduce the non-perturbative behaviour. For instance, models based on quasi-particle pictures with effectively modified properties due to strong interactions are successful in describing lattice QCD results [14–23]. Analytical approaches with a rigorous link to QCD (cf. [24] for a survey) such as direct HTL resummation [25–28] or the Φ-functional approach [29–34] formulated in terms of dressed propagators are successful in describing lattice QCD on temperatures $T \gtrsim 2T_c$.

It is the aim of the present paper to show the successful applicability of our quasi-particle model (QPM) for describing lattice QCD results and to motivate the model starting from the Φ-functional approach to QCD. In Sect. 2, we review the QPM and compare with recent lattice QCD results for pressure and entropy density. In Sect. 3, a possible chain of approximations is outlined starting from QCD within the Φ-functional approximation scheme which motivates our formulation of QCD thermodynamics in terms of quasi-particle excitations. We summarize our results in Sect. 4.

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2 QPM and comparison with lattice QCD

In our model, the pressure p for $N_f = 2$ light quark flavours in thermal equilibrium as a function of temperature T and one chemical potential μ_q ($\mu_g = 0$) reads

$$p(T,\mu_q) = \sum_{a=q,g} p_a - B(T,\mu_q)\,, \tag{1}$$

where $p_a = d_a/(6\pi^2)\int_0^\infty \mathrm{d}k k^4 \left(f_a^+ + f_a^-\right)/\omega_a$ denote the partial pressures of quarks (q) and transverse gluons (g). Here, $d_q = 2N_f N_c$, $d_g = N_c^2 - 1$, $N_c = 3$, and $f_a^\pm = (\exp([\omega_a \mp \mu_a]/T) + S_a)^{-1}$ with $S_q = 1$ for fermions and $S_g = -1$ for bosons. $B(T,\mu_q)$ is determined from thermodynamic self-consistency and the stationarity of p under functional variation with respect to the self-energies, $\delta p/\delta \Pi_a = 0$ [35]. The Π_a enter the quasi-particle dispersion relations, ω_a being approximated by asymptotic mass shell expressions near the light cone, $\omega_a = \sqrt{k^2 + \Pi_a}$. We employ the asymptotic expressions of the gauge independent hard thermal (dense) loop self-energies [36]. Finite bare quark masses $m_{0;q}$ as used in lattice simulations can be implemented following [37, 38].

By replacing the running coupling g^2 in Π_a with an effective coupling $G^2(T,\mu_q)$, non-perturbative effects in the vicinity of T_c are accomodated. In this way, we achieve enough flexibility to describe lattice QCD results. We parametrize $G^2(T,\mu_q = 0)$ [39] by

$$G^2(T,\mu_q = 0) = \begin{cases} G^2_{(2)}(\xi(T)),\ T \geq T_c, \\ G^2_{(2)}(\xi(T_c)) + b\left(1 - \frac{T}{T_c}\right),\ T < T_c, \end{cases} \tag{2}$$

where $G^2_{(2)}$ is the relevant part of the two-loop running coupling and $\xi(T) = \lambda(T - T_s)/T_c$ contains a scale parameter λ and an infrared regulator T_s. The effective coupling G^2 for arbitrary T and μ_q can be found by solving a quasi-linear partial differential equation which follows from Maxwell's relation,

$$a_{\mu_q} \frac{\partial G^2}{\partial \mu_q} + a_T \frac{\partial G^2}{\partial T} = b\,. \tag{3}$$

The coefficients in (3) explicitly read (neglecting for simplicity additional contributions stemming from the T-dependent bare quark masses as employed in lattice simulations)

$$a_T = I_1 \frac{C_f}{4}\left(T^2 + \frac{\mu_q^2}{\pi^2}\right), \tag{4}$$

$$a_{\mu_q} = -I_2 \frac{C_f}{4}\left(T^2 + \frac{\mu_q^2}{\pi^2}\right) - I_3\left(\left[N_c + \frac{N_f}{2}\right]\frac{T^2}{6} + \frac{N_c N_f}{12\pi^2}\mu_q^2\right), \tag{5}$$

$$b = -I_1 \frac{C_f}{2} T G^2 + I_2 \frac{C_f}{2}\frac{\mu_q}{\pi^2} G^2 + I_3 \frac{N_c N_f}{6\pi^2}\mu_q G^2\,. \tag{6}$$

Here,

$$I_1 = \frac{d_q}{4\pi^2 T}\int_0^\infty \mathrm{d}k \frac{k^2}{\omega_q}\left(\mathrm{e}^{\omega_q^+/T}(f_q^-)^2 - \mathrm{e}^{\omega_q^-/T}(f_q^+)^2\right), \tag{7}$$

$$I_2 = \frac{d_q}{2\pi^2 T}\int_0^\infty \mathrm{d}k \frac{k^2}{\omega_q}\left(f_q^+ - \frac{L_q f_q^+}{\omega_q}\left[\frac{1}{\omega_q} + \frac{\mathrm{e}^{\omega_q^-/T}}{T} f_q^+\right] + \frac{\mu_q}{2T}\mathrm{e}^{\omega_q^-/T}(f_q^+)^2 + f_q^- - \frac{L_q f_q^-}{\omega_q}\left[\frac{1}{\omega_q} + \frac{\mathrm{e}^{\omega_q^+/T}}{T} f_q^-\right] - \frac{\mu_q}{2T}\mathrm{e}^{\omega_q^+/T}(f_q^-)^2\right), \tag{8}$$

$$I_3 = \frac{d_g}{\pi^2 T}\int_0^\infty \mathrm{d}k \frac{k^2}{\omega_g} f_g\left(1 - \frac{L_g}{\omega_g}\left[\frac{1}{\omega_g} + \frac{\mathrm{e}^{\omega_g/T}}{T} f_g\right]\right), \tag{9}$$

with $f_g^\pm \equiv f_g$, $\omega_q^\pm = \omega_q \pm \mu_q$, $L_a = 2k^2/3 + \Pi_a/2$ and $C_f = (N_c^2 - 1)/2N_c$.

The entropy density $s = \partial p/\partial T = \sum_{a=q,g} s_a$ and the net density $n = n_q = \partial p/\partial \mu_q$ follow from (1) as follows:

$$s_a = \frac{d_a}{2\pi^2 T}\int_0^\infty \mathrm{d}k k^2\left(\frac{\frac{4}{3}k^2 + \Pi_a}{\omega_a}\left[f_a^+ + f_a^-\right] - \mu_a\left[f_a^+ - f_a^-\right]\right), \tag{10}$$

$$n_q = \frac{d_q}{2\pi^2}\int_0^\infty \mathrm{d}k k^2 \left[f_q^+ - f_q^-\right]. \tag{11}$$

In Fig. 1, we exhibit QPM results for p and s at $\mu_q = 0$ compared with lattice QCD results for different numbers of quark flavours [40, 41].

Recently, the decomposition of p into a Taylor series in powers of μ_q/T for small μ_q was studied in lattice QCD [43, 44],

$$p(T, \mu_q) = T^4 \sum_{n=0}^\infty c_n(T)\left(\frac{\mu_q}{T}\right)^n. \tag{12}$$

The expansion coefficients $c_n(T)$, vanishing for odd n and depending only on the temperature T, follow using (1) from

$$c_n(T) = \frac{1}{n!}\frac{\partial^n (p/T^4)}{\partial(\mu_q/T)^n}\bigg|_{\mu_q=0}. \tag{13}$$

$c_n(T)$ depend on G^2 and its derivatives with respect to μ_q at $\mu_q = 0$, thus testing (3). Furthermore, the net density n can also be decomposed into a Taylor series at small μ_q with expansion coefficients $c_n(T)$. Therefore, the higher order coefficients $c_{2,4,6}(T)$ serve for a more direct test of the

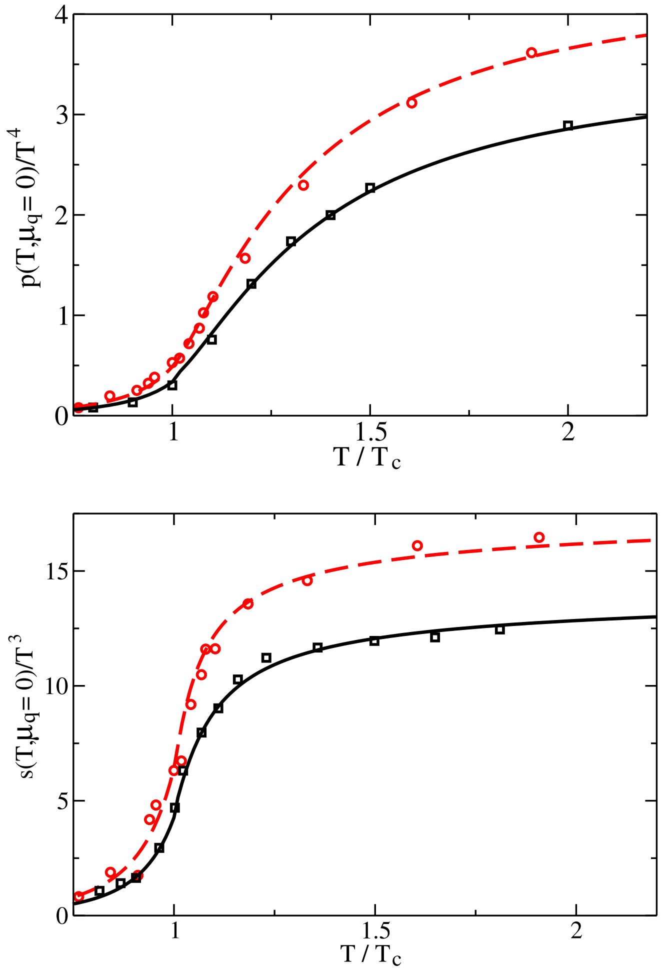


Fig. 1. Comparison of our QPM with lattice QCD results (*symbols*) for $p(T, \mu_q = 0)/T^4$ (*upper panel*) and $s(T, \mu_q = 0)/T^3$ (*lower panel*) as functions of T/T_c for $N_f = 2$ (*squares*) [40] and $N_f = 2+1$ ($\mu_s = 0$) (*circles*) [40, 41]. Raw lattice QCD data are continuum extrapolated as advocated in [40, 42]. QPM parameters: $\lambda = 4.4$, $T_s = 0.67 T_c$, $b = 344.4$, $B(T_c) = 0.31 T_c^4$ with $T_c = 175$ MeV for $N_f = 2$ and $\lambda = 7.6$, $T_s = 0.80 T_c$, $b = 348.7$, $B(T_c) = 0.52 T_c^4$ with $T_c = 170$ MeV for $N_f = 2+1$

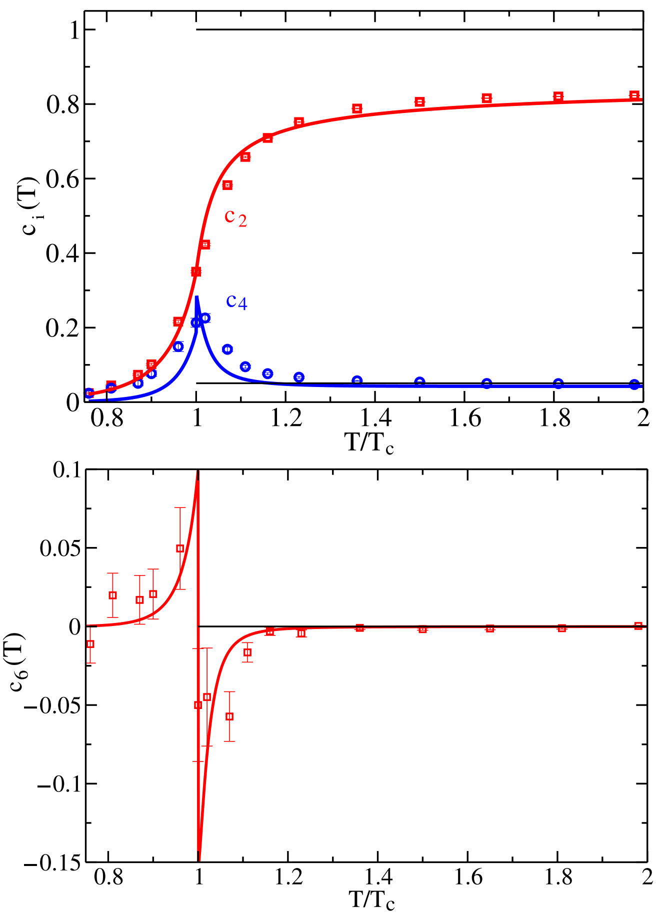


Fig. 2. Comparison of our QPM with lattice QCD results (*symbols*) [43, 44] for $c_{2,4}(T)$ (*upper panel*) and $c_6(T)$ (*lower panel*) as functions of T/T_c for $N_f = 2$. QPM parameters: $\lambda = 12.0$, $T_s = 0.87T_c$, $b = 426.1$, with $T_c = 175$ MeV. The *horizontal lines* at $T \geq T_c$ depict the corresponding Stefan–Boltzmann values highlighting the effects of the strong interaction near T_c

applicability of our model at finite μ_q. In Fig. 2, we compare $c_{2,4,6}(T)$ evaluated from (13) with lattice QCD results for $N_f = 2$ [43, 44]. In particular, the pronounced structures in the vicinity of T_c are fairly well reproduced [39].

3 Foundations of the QPM

Having successfully reproduced first-principle lattice QCD results, it would be desirable to establish contact between our ad hoc introduced QPM in Sect. 2 and QCD as the fundamental microscopic gauge field theory of strong interactions. In order to motivate our quasi-particle model, we present a possible chain of approximations starting from QCD within the Φ-functional approach following the pioneering work of [29–34]. We concentrate on the entropy density s and the net density n, as these turn out to possess a simple structure supporting the picture of quasi-particle excitations. Other thermodynamic quantities such as the pressure p or the energy density e are determined from s and n. Although rather strong assumptions become mandatory in the derivation, one should be aware of the remarkable success of our QPM in describing lattice QCD results.

In the Φ-functional approach [45, 46] to QCD, the thermodynamic potential $\Omega = -T \ln Z$ can be expressed as a functional of dressed propagators of gluons, D, quarks, S and Faddeev–Popov ghost fields, G,

$$\frac{\Omega[D,S,G]}{T} = \frac{1}{2}\mathrm{Tr}[\ln D^{-1} - \Pi D] - \mathrm{Tr}[\ln S^{-1} - \Sigma S] - \mathrm{Tr}[\ln G^{-1} - \Xi G] + \Phi[D,S,G]\,. \qquad (14)$$

Here, ghost field contributions compensate for possible unphysical degrees of freedom in the gluon propagator. While the propagators in (14) depend on the specific gauge, $\Omega = -pV$ must be gauge independent. For convenience, we choose the Coulomb gauge in the following, in which ghost fields do not propagate and the gluon propagator consists only of the physical transverse and longitudinal modes. The functional $\Phi[D,S]$ is given by the infinite sum of all two-particle irreducible skeleton diagrams constructed from D and S.

The self-energies are related to the dressed propagators by Dyson's equations:

$$\Pi[D] = D^{-1} - D_0^{-1}, \quad \Sigma[S] = S^{-1} - S_0^{-1}\,, \qquad (15)$$

where D_0 and S_0 represent the bare propagators of gluon and quark fields, respectively. Demanding the stationarity of Ω under functional variation with respect to the dressed propagators [47]

$$\left.\frac{\delta\Omega[D,S]}{\delta D}\right|_{D_0} = \left.\frac{\delta\Omega[D,S]}{\delta S}\right|_{S_0} = 0\,, \qquad (16)$$

the self-energies follow self-consistently by cutting a dressed propagator line in Φ resulting in the gap equations

$$\Pi = 2\frac{\delta\Phi[D,S]}{\delta D}, \quad \Sigma = -\frac{\delta\Phi[D,S]}{\delta S}\,. \qquad (17)$$

The trace "Tr" in (14) has to be taken over all states of the relativistic many-particle system. In the imaginary time formalism it can be rewritten in the form $\mathrm{Tr} \to \mathrm{tr}\, \beta V T \sum_{n=-\infty}^{+\infty} \int \mathrm{d}^3k/(2\pi)^3$. Here, V is the volume of the system, $\beta = 1/T$ and "tr" denotes the remaining trace over the occurring discrete indices including colour, flavour, Lorentz or spinor indices. Introducing the four-momentum $k^\nu = (\omega, k) = (\mathrm{i}\omega_n, k)$, the sums have to be taken over the Matsubara frequencies $\omega_n = 2n\pi T$ (or $(2n+1)\pi T - \mathrm{i}\mu$) for gluons (or quarks). They can be evaluated by using standard contour integration techniques in the complex ω-plane [36, 48] wrapping up the poles of the propagators. Expressing the analytic propagators in terms of their spectral densities ρ, one can define

$$\rho_{D(S)}(\omega, |k|) = 2 \lim_{\epsilon \to 0} \mathrm{Im}\, D(S)(\omega + \mathrm{i}\epsilon, |k|) \qquad (18)$$

for real ω. Similarly, the imaginary parts of functions of the analytic propagators obeying the same pole structures can be defined. Hence, Ω reads with retarded propagators D and S depending on ω and $k = |\boldsymbol{k}|$

$$\begin{aligned}\frac{\Omega[D,S]}{V} &= \mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4} n(\omega)\, \mathrm{Im}[\ln D^{-1} - \Pi\, D] \qquad (19)\\ &+ 2\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4} f(\omega)\, \mathrm{Im}[\ln S^{-1} - \Sigma\, S]\\ &+ \frac{T}{V}\Phi[D,S]\,,\end{aligned}$$

where $\int \mathrm{d}^4k = \int \mathrm{d}^3k \int \mathrm{d}\omega$, and $n(\omega) = (\mathrm{e}^{\beta\omega}-1)^{-1}$ ($f(\omega) = (\mathrm{e}^{\beta(\omega-\mu)}+1)^{-1}$) denotes the statistical distribution function for gluons (quarks with chemical potential μ).

Due to the stationarity property (16), the entropy density $s = -\partial(\Omega/V)/\partial T$ and the net density $n = -\partial(\Omega/V)/\partial\mu$ contain only explicit temperature and chemical potential derivatives of $n(\omega)$ and $f(\omega)$, although the propagators in (19) depend implicitly on T and μ through their spectral densities. Using $\mathrm{Im}\,(\Pi D) = \mathrm{Im}\,\Pi\,\mathrm{Re}\,D + \mathrm{Re}\,\Pi\,\mathrm{Im}\,D$, one finds for the entropy density $s = s_g + s_q + s'$, with

$$s_g = -\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial n(\omega)}{\partial T}[\mathrm{Im}\ln D^{-1} - \mathrm{Im}\,\Pi\,\mathrm{Re}D]\,, \qquad (20)$$

$$s_q = -2\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial f(\omega)}{\partial T}[\mathrm{Im}\ln S^{-1} - \mathrm{Im}\,\Sigma\,\mathrm{Re}S]\,, \qquad (21)$$

$$\begin{aligned}s' = &-\left.\frac{\partial\left(\frac{T}{V}\Phi[D,S]\right)}{\partial T}\right|_{D,S} + 2\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial f(\omega)}{\partial T}\mathrm{Re}\Sigma\,\mathrm{Im}S\\ &+ \mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial n(\omega)}{\partial T}\mathrm{Re}\Pi\,\mathrm{Im}D\,. \qquad (22)\end{aligned}$$

Similarly, for the net density one finds $n = n_q + n'$, with

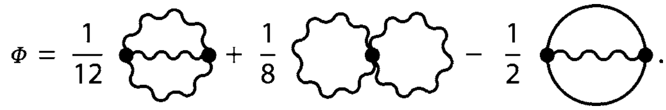
$$n_q = -2\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial f(\omega)}{\partial \mu}[\mathrm{Im}\ln S^{-1} - \mathrm{Im}\,\Sigma\,\mathrm{Re}S], \qquad (23)$$

$$n' = 2\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial f(\omega)}{\partial \mu}\mathrm{Re}\Sigma\,\mathrm{Im}S - \left.\frac{\partial\left(\frac{T}{V}\Phi[D,S]\right)}{\partial \mu}\right|_{D,S}. \qquad (24)$$

While the sum integrals in Ω (19) contain ultraviolet divergencies which must be regularized, the expressions for s_g, s_q and n_q in (20), (21) and (23) are manifestly ultraviolet convergent, because the derivatives of the statistical distribution functions vanish for $\omega \to \pm\infty$. In addition, introducing real multiplicative renormalization factors for propagators and self-energies, these factors simply drop out of s and n.

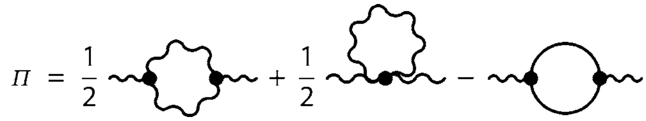
Self-consistent (or Φ-derivable) approximation schemes preserve the stationarity property (16) of Ω when truncating the infinite sum in Φ at a specific loop order, while corresponding self-energies and propagators are self-consistently evaluated from (17) and Dyson's equations. Nevertheless, self-consistency does not guarantee gauge invariance which is an important issue in truncated expansion schemes. In fact, by modifying propagators but leaving vertices unaffected, the Ward identities are violated.

We consider Φ at two-loop order in the following, which is diagrammatically represented by [49, 50]



(25)

Here, wiggly (solid) lines denote gluons (quarks). The self-consistent self-energies are accordingly



(26)

(27)

Although vertex corrections can be implemented self-consistently [51], they turn out to be negligible at two-loop order in Φ [33]. In addition, $s' = n' = 0$ is found for the residual contributions of entropy density and net density in (22) and (24) at two-loop order [33]. This topological feature, being related to (17), has also been observed in massless Φ^4-theory [49, 50, 52] and in QED [53, 54].

Concentrating on the gluonic contribution s_g, (20) can be rewritten by using the identity

$$\begin{aligned}\mathrm{Im}[\ln D^{-1}(\omega,k)] = &-\pi\,\mathrm{sgn}(\omega)\,\Theta(-\mathrm{Re}D^{-1}(\omega,k)) \qquad (28)\\ &+ \arctan\left(\frac{\mathrm{Im}\Pi(\omega,k)}{\mathrm{Re}D^{-1}(\omega,k)}\right)\end{aligned}$$

where $-\pi/2 < \arctan x < \pi/2$. Hence, s_g can be decomposed into $s_g = s_{g,\mathrm{QP}} + s_{g,\mathrm{LD}}$, with

$$s_{g,\mathrm{QP}} = \mathrm{tr}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\int\frac{\mathrm{d}\omega}{2}\frac{\partial n(\omega)}{\partial T}\mathrm{sgn}(\omega)\,\Theta(-\mathrm{Re}D^{-1})\,, \qquad (29)$$

$$\begin{aligned}s_{g,\mathrm{LD}} = &\,\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial n(\omega)}{\partial T}\\ &\times\left\{\mathrm{Im}\Pi\,\mathrm{Re}D - \arctan\left(\frac{\mathrm{Im}\Pi}{\mathrm{Re}D^{-1}}\right)\right\}. \qquad (30)\end{aligned}$$

Here, (29) accounts for the contribution of the dynamical quasi-particles to s_g defined by the poles of D, and (30) represents the contribution from the continuum part of the spectral density associated with a cut below the light cone $|\omega| < k$ [37, 38, 55, 56] representing Landau damping. Applying a similar identity for $\mathrm{Im}[\ln S^{-1}(\omega,k)]$, s_q and n_q in (21) and (23) can be decomposed similarly into quasi-particle and Landau damping contributions.

In Coulomb gauge, D consists of a longitudinal and a transverse part, D_L and D_T. Similarly, the (massless) quark propagator consists of two different branches with chirality either equal (positive energy states) or opposite (negative energy states) to the helicity. By employing the gauge invariant hard thermal loop (HTL) expressions $\hat{\Pi}$ ($\hat{\Sigma}$) for the gluon (quark) self-energies in the following, one obtains gauge invariant approximations of s and n. The HTL expressions read [36]

$$\hat{\Pi}_\mathrm{L}(\omega,k) = \hat{m}_\mathrm{D}^2\left(1-\frac{\omega}{2k}\ln\frac{\omega+k}{\omega-k}\right), \tag{31}$$

$$\hat{\Pi}_\mathrm{T}(\omega,k) = \frac{1}{2}\left(\hat{m}_\mathrm{D}^2+\frac{\omega^2-k^2}{k^2}\hat{\Pi}_\mathrm{L}(\omega,k)\right), \tag{32}$$

$$\hat{\Sigma}_\pm(\omega,k) = \frac{\hat{M}^2}{k}\left(1-\frac{\omega\mp k}{2k}\ln\frac{\omega+k}{\omega-k}\right), \tag{33}$$

with Debye screening mass (allowing, in general, for different chemical potentials μ_i)

$$\hat{m}_\mathrm{D}^2 = \left([2N_c+N_f]T^2+N_c\sum_i\frac{\mu_i^2}{\pi^2}\right)\frac{g^2}{6}, \tag{34}$$

long-wavelength fermionic frequency

$$\hat{M}^2 = \frac{N_c^2-1}{16N_c}\left(T^2+\frac{\mu_i^2}{\pi^2}\right)g^2, \tag{35}$$

and running coupling g^2. Although being derived originally for soft external momenta $\omega, k \sim gT \ll T$, they coincide on the light cone with complete one-loop results [57] as exhibited in Fig. 3. Finite quark masses, $m < T$, turn out to be negligible. The corresponding propagators are evaluated from Dyson's equations.

For $k \sim T, \mu$ the poles of both, longitudinal gluon propagator as well as abnormal fermion branch have exponentially vanishing residues [37, 38] giving only minor contributions to the thermodynamics. Therefore, we assume that these collective modes can be neglected in the following. Furthermore, being a severe approximation, we also neglect any imaginary parts of the self-energies, i.e. $\mathrm{Im}\hat{\Pi}_\mathrm{T} = \mathrm{Im}\hat{\Sigma}_+ = 0$. Then the Landau damping contributions to s_g, s_q and n_q vanish. Finite width effects associated with imaginary parts of the self-energies are discussed by Peshier [55, 56]. Including Landau damping as well as the exponentially suppressed modes, it was shown in [17] that in this way some ambiguities arising when solving (3) can be eliminated.

Performing the ω-integration in (29) (but now for $\hat{D}_\mathrm{T}$), the only contributions stem from $\omega^2 \geq \omega_T^2$ because of the Θ-function, where ω_T is the positive solution of $\omega^2 - k^2 - \hat{\Pi}_\mathrm{T}(\omega,k) = 0$. Therefore, the ω-integral in (29) reads

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2}\frac{\partial n(\omega)}{\partial T}\mathrm{sgn}(\omega)\,\Theta\left(-\mathrm{Re}\hat{D}_\mathrm{T}^{-1}\right) = \int_{\infty}^{\omega_T}\frac{\mathrm{d}\omega}{2}\left(\frac{\partial n(-\omega)}{\partial T}-\frac{\partial n(\omega)}{\partial T}\right). \tag{36}$$

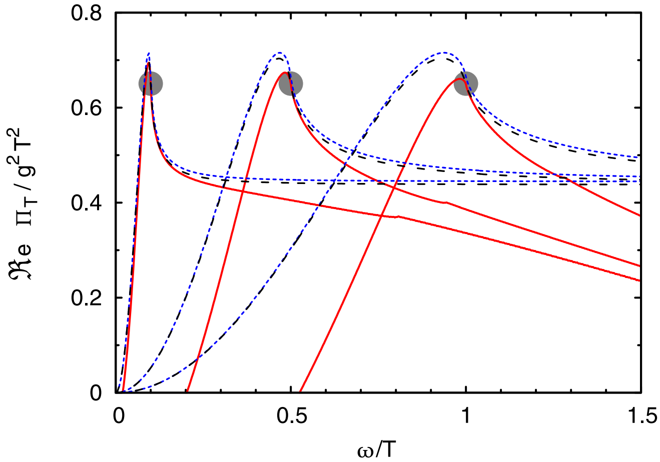


Fig. 3. Comparison of HTL approximation (the nearly indistinguishable short and long *dashed curves* are for massless (32) (cf. [57]) and massive quarks, $m_q = 0.4\,T$, as used in lattice calculations [40–44], respectively) with one-loop results (*solid curves* for massive quarks) [58] of the scaled real part of the transverse gluon self-energy as function of ω/T for $k/T = 0.1$, 0.5 and 1 from left to right. $N_f = 2$, $\mu_i = 0$. In the vicinity of the light cone $\omega = k$ (*shaded regions*), HTL results approximate one-loop results fairly well even for $k \sim T$, which is the thermodynamically relevant part in momentum space but with decreasing agreement for increasing k

The remaining integration is performed through an integration by parts using $-\partial n(\omega)/\partial T = \partial n(-\omega)/\partial T = \partial\sigma(\omega)/\partial\omega$ for the spectral function $\sigma(\omega) = -n(\omega)\ln n(\omega) + (1+n(\omega))\ln(1+n(\omega))$. Taking the trace over polarization and colour degrees of freedom for the transverse gluon modes, one finds

$$s_{g,\mathrm{QP}} = -2\left(N_c^2-1\right)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left(\ln(1-\mathrm{e}^{-\beta\omega_T})-\frac{\beta\omega_T}{\mathrm{e}^{\beta\omega_T}-1}\right). \tag{37}$$

Similarly, $s_{q,\mathrm{QP}}$ can be evaluated, where non-vanishing contributions to the ω-integration stem from $\omega \geq \omega_+$. Here, ω_+ is the solution of $\omega - k - \hat{\Sigma}_+(\omega,k) = 0$ for the positive fermion branch. Using $-\partial f(\omega)/\partial T = \partial\sigma(\omega)/\partial\omega$ for the spectral function $\sigma(\omega) = -f(\omega)\ln f(\omega) - (1-f(\omega))\ln(1-f(\omega))$, the ω-integral can be integrated by parts. Antiquarks are included by simply replacing $\mu \to -\mu$ in $f(\omega)$. Taking the trace over remaining spin, colour and flavour degrees of freedom, one finds

$$s_{q,\mathrm{QP}} = 2N_cN_f\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left(\ln(1+\mathrm{e}^{-\beta(\omega_+-\mu)})+\frac{\beta(\omega_+-\mu)}{\mathrm{e}^{\beta(\omega_+-\mu)}+1}\right) + 2N_cN_f\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left(\ln(1+\mathrm{e}^{-\beta(\omega_++\mu)})+\frac{\beta(\omega_++\mu)}{\mathrm{e}^{\beta(\omega_++\mu)}+1}\right). \tag{38}$$

$s_{g,\mathrm{QP}}$ and $s_{q,\mathrm{QP}}$ in (37) and (38) represent the entropy density contributions of non-interacting quasi-particles with quantum numbers of transverse gluons (quarks) and dispersion relation ω_T (ω_+).

Correspondingly, $n_{q,\mathrm{QP}}$ is evaluated using $-\partial f(\omega)/\partial\mu = \partial f(\omega)/\partial\omega$. Adding antiquarks by $\mu \to -\mu$ in $f(\omega)$ (note that now $\partial f(\omega)/\partial\mu = \partial f(\omega)/\partial\omega$) and taking the trace, $n_{q,\mathrm{QP}}$ reads

$$n_{q,\mathrm{QP}} = 2N_c N_f \int \frac{\mathrm{d}^3k}{(2\pi)^3}\left(\frac{1}{\mathrm{e}^{\beta(\omega_+-\mu)}+1} - \frac{1}{\mathrm{e}^{\beta(\omega_++\mu)}+1}\right). \tag{39}$$

Finally, we approximate the quasi-particle dispersion relations by the asymptotic mass shell expressions near the light cone, thus neglecting any momentum or energy dependence of the self-energies. We employ $\omega_T \to \omega_g = \sqrt{k^2 + \Pi_g}$ and $\omega_+ \to \omega_q = \sqrt{k^2 + \Pi_q}$ as in Sect. 2 with asymptotic masses $\Pi_g = \hat{m}_{\mathrm{D}}^2/2$ and $\Pi_q = 2\hat{M}^2$, considering only one chemical potential $\mu_i = \mu_q$ ($\sum_i \mu_i^2 \to N_f \mu_q^2$). Integrating the logarithmic terms in (37) and (38) by parts (note that (39) already obeys the desired form), one exactly recovers the expressions (10) and (11) of our QPM, where the replacement of g^2 by G^2 remains as phenomenological procedure on top of the listed "approximations".

4 Conclusions

In summary, motivated by the successful reproduction of available results of QCD thermodynamics, we attempted to make a collection of necessary steps to establish the link of our employed quasi-particle model to QCD. Quite severe assumptions had to be made. Even with these, resulting in the formal structure of our model, an additional and crucial point is the parametrization of the effective coupling. While allowing for an accurate two-parameter fit of many different lattice QCD results, it requires a foundation. In this respect, we refer to the work in [59], where the authors argue that the pure quasi-particle excitations, deduced from a preliminary study of the poles of quark and gluon propagators [60], are too heavy to saturate the pressure delivered from lattice calculations [41], i.e. signalling the necessity of including additional degrees of freedom. Further systematic studies of the relevant degrees of freedom in the strongly coupled quark–gluon medium near T_c are eagerly awaited to have some guidance.

An additional issue is the chiral extrapolation. The quark masses of $0.4\,T$ employed in the lattice simulations here analyzed correspond to unphysically heavy pions with $m_\pi \sim 770$ MeV. In the one-loop and HTL gluon self-energy considered here, such a finite quark mass has a tiny (negligible) impact. A more rigorous treatment of finite quark mass effects must be accomplished to arrive at a suitable chiral extrapolation.

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